Parity Effect and Tunnel Magnetoresistance of Ferromagnet / Superconductor / Ferromagnet Single-Electron Tunneling Transistors

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We theoretically study the tunnel magnetoresistance(TMR) of ferromagnet / superconductor / ferromagnet single-electron tunneling transistors with a special attention to the parity effect. It is shown that in the plateau region, there is no spin accumulation in the island even at finite bias voltage. However, the information of the injected spin is carried by the excess electron and thus the TMR exists. The spin relaxation rate of the excess electron can be estimated from the TMR. We also show that the TMR increases with decreasing the size of the superconducting island.

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Single-electron tunneling(SET) transistor is a key device of "single electronics" since the transfer of a single electron can be controlled by gate and bias voltages [1, 2]. The important quantity of the SET transistor is the electrostatic energy of excess electrons in the island called "charging energy", which has a significant effect on charge transport, *i. e.* , the Coulomb blockade(CB) [1]. In the CB region, sequential tunneling, where tunneling events in each junction occur independently, is blocked at $T = 0$ due to the increase of the charging energy. Recently much attention has been devoted to the SET transistor with a superconducting island and normal conducting electrodes(N/S/N) [3, 4, 5, 6, 7, 8]. In a superconducting island, where Cooper pairs form the condensate, the addition of one extra electron costs the superconducting gap energy Δ. This leads to the parity effect: physical properties of the system depend on the parity of the electron number in the island. In SET transistors consisting of normal conducting islands and electrodes(N/N/N) the current depends e periodically on the gate charge. However, the tunneling current of the N/S/N SET transistor is expected to be $2e$ periodic in the gate charge below the crossover temperature due to the parity effect[3]. The clear signature of $2e$ periodicity for the N/S/N SET transistor was observed[6, 7].

The research on the N/S/N SET transistor has focused primary on the charge degrees of freedom of electrons, by contrast, its spin degrees of freedom have not yet received much attention. However, an increasing number of researches on spin-electronics show that the spin of electron offers unique possibilities for finding novel mechanisms for future spin-electronic devices [9, 10]. The spin-polarized current injected from a ferromagnetic(F) electrode into the N or S island gives rise to a nonequilibrium spin density in the island. In the F/N/F SET transistor, the tunnel magnetoresistance(TMR) is brought about by spin accumulation in the island [11, 12, 13, 14]. Recently Takahashi *et al.* have studied the magnetoresistive effects of the F/S/F double tunnel junction caused by competition between superconductivity and spin accumulation [15]. They have shown that the F/S/F double tunnel junction is a magnetoresistive device to control superconductivity by applying the bias voltage or current. The suppression of the superconductivity by the spin injection in the double tunnel junction was observed by Dong *et al.* [16].

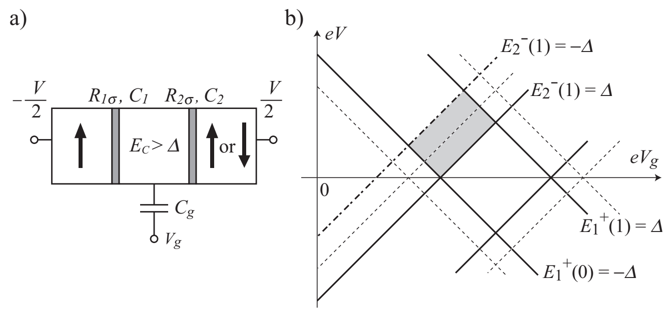

FIG. 1: a)Schematic diagram of a F/S/F SET transistor. The arrows indicate the magnetizations of the left and right electrodes. b) The gate and bias voltage diagram of a symmetric SET transistors. Solid lines indicate the boundaries for the F/S/F SET transistor. Thin dashed lines are the boundaries for the F/N/F SET transistor ($\Delta = 0$). The dot-dashed line indicates a boundary of the plateau region.

In this article, we theoretically study the spin-dependent transport of the F/S/F SET transistors at zero temperature with a special attention to the parity effect. We assume that the charging energy E_C is larger than Δ and neglect the two-electron tunneling [17] which becomes important in the opposite situation ($E_C < \Delta$). In such a system, we have the special region called "plateau region" where the tunneling current is dominated by the transition rate from the odd-state to the even-state [3]. We show that in the plateau region, spin accumulation is forbidden by the superconducting gap even at finite bias voltage. However, the information of the injected spin is carried by the excess electron and the TMR exists. The spin relaxation rate of the excess electron can be estimated from the TMR. We also show that the TMR increases as the size of the island decreases.

We consider the F/S/F SET transistor shown in Fig. 1 (a). For simplicity we assume that the insulating barriers of junctions 1(left) and 2(right) are the same; we subsequently set $C_1 = C_2 \equiv C$. We also assume that the

left and right electrodes are made of the same material with the spin polarization P.

The Hamiltonian of the SET transistor is given by

$$H = H_L + H_I + H_R + H_{ch} + H_{TL} + H_{TR}, \qquad (1)$$

where H_L, H_I, and H_R are the Hamiltonians of the left electrode, the central island, and the right electrode, respectively. The Hamiltonian H_{ch} represents the charging energy and the tunneling processes are described by

$$H_{TL(R)} = \sum_{k,q,\sigma} T_{kq}^{L(R)} c_{k\sigma}^{\dagger} c_{q\sigma} + \text{h.c.}, \qquad (2)$$

where the subscript k indicates the wave vector in the left(right) electrode while q represents that in the island and no spin flip is assumed in the tunneling process.

The energy change due to the forward tunneling of an electron with spin σ through the junctions 1 and 2 are respectively given by

$$E_1^+(n) \;=\; (1+2n)E_c - \frac{C_g}{C_\Sigma} eV_g - \frac{1}{2}eV \qquad (3)$$

$$E_2^-(n) \;=\; (1-2n)E_c + \frac{C_g}{C_\Sigma} eV_g - \frac{1}{2}eV, \qquad (4)$$

where n is the number of excess electrons of the initial state in the island, $C_\Sigma \equiv 2C + C_g$, and the superscripts $\pm$ implies that the number of excess electrons in the final state is $n \pm 1$. The energy changes for the backward tunneling are given by $E_1^-(n) = E_2^-(n) + eV$ and $E_2^+(n) = E_1^+(n) + eV$.

In ordinary tunnel junctions with negligible charging effect, individual electrons are unstable for the forward tunneling across the junction, and lower their energy by the bias potential $(-eV/2 < 0)$. In SET transistors, however, there are special regions called CB region in the gate and bias voltage diagram, where the charge state of the island with n excess electrons is stable with respect to tunneling. For the symmetric SET transistor with a normal conducting island, the CB regions are the rhombuses determined by $E_1^+(n), E_1^-(n), E_2^+(n)$, and $E_2^-(n) \geq 0$. The boundaries of CB regions are indicated by thin dashed lines in Fig. 1 (b).

For the superconducting island, however, it is know that these boundaries are modified by superconductivity. In order to obtain the modified CB regions, we evaluate the tunneling rates following Fazio and Schön [3]. Since the island is in the superconducting state, it is convenient to rewrite the electron operators in the island in terms of the quasiparticle operators by using the Bogoliubov transformations. Then the transition rates are determined by using the Golden-rule arguments [3]. Let us first consider an electron tunnels from the left electrode into the island with $n = 0$, thereby changing the electron number from $n = 0$ to $n = 1$ with the rate

$$\Gamma_{1\sigma}^+(0) = \frac{1}{e^2 R_{1\sigma}} \int_{-\infty}^{\infty} dE \int_{-\infty}^{\infty} dE' \mathcal{D}(E') \times f(E)\left[1 - f(E')\right] \delta(E' - E + E_1^+(0)), \qquad (5)$$

where $\mathcal{D}(E) \equiv |E|/\sqrt{E^2 - \Delta^2}$ is the normalized BCS density of states and $f(E)$ is the Fermi distribution function. The tunnel conductance of the junction is defined as $R_{1\sigma}^{-1} \equiv (4\pi e^2/\hbar) N_\sigma^I N_\sigma^L |T|^2$, where the tunnel matrix elements T_{kq}^L and T_{kq}^R are considered as a constant T and $N_\sigma^{I(L)}$ denotes the density of states of the island in the normal conducting state(left electrode). Since we assume the temperature is zero, the Fermi distribution function can be replaced by the step function and we have

$$\Gamma_{1\sigma}^+(0) = \begin{cases} 0 & (E_1^+(0) \geq -\Delta) \\ \dfrac{\sqrt{E_1^+(0)^2 - \Delta^2}}{e^2 R_{1\sigma}} & (E_1^+(0) < -\Delta) \end{cases}. \qquad (6)$$

Therefore, the boundary of the CB region for $n = 0$ given by $E_1^+(0) = 0$ is lifted to the solid line determined by $E_1^+(0) = -\Delta$ as shown in Fig. 1 (b). The other three boundaries also shift outward and the CB region is determined by $E_1^+(0), E_1^-(0), E_2^+(0)$, and $E_2^-(0) \geq -\Delta$.

Next we consider an electron tunnels from the island with $n = 1$ to the right electrode, thereby changing the electron number from $n = 1$ to $n = 0$ with the rate

$$\Gamma_2^-(1) = \frac{1}{e^2 R_{2\sigma}} \int_{-\infty}^{\infty} dE \int_{-\infty}^{\infty} dE' \mathcal{D}(E) \times f(E - \delta\mu)\left[1 - f(E')\right] \delta(E' - E + E_2^-(1)), \qquad (7)$$

where $R_{2\sigma}^{-1} = (4\pi e^2/\hbar) N_\sigma^I N_\sigma^R |T|^2$ and $\delta\mu$ is the shift of the chemical potential fixed by the constraint of one excess electron charge in the island [3]. The shift of the chemical potential is $\delta\mu = \Delta$ and we have

$$\Gamma_{2\sigma}^-(1) = \begin{cases} 0 & (E_2^-(1) \geq \Delta) \\ \dfrac{d}{e^2 R_{2\sigma}} & (|E_2^-(1)| < \Delta) \\ \dfrac{d}{e^2 R_{2\sigma}} + \dfrac{\sqrt{E_2^-(1)^2 - \Delta^2}}{e^2 R_{2\sigma}} & (E_2^-(1) \leq -\Delta) \end{cases}, \qquad (8)$$

where $d = 1/N_\sigma^I$ is the average level spacing of the island. Therefore, the boundary of the CB region for $n = 1$ given by $E_2^-(1) = 0$ is also moved to the solid line determined by $E_2^-(1) = \Delta$ as shown in Fig. 1 (b). The other three boundaries also shift inward and the CB region is determined by $E_1^+(1), E_1^-(1), E_2^+(1)$, and $E_2^-(1) \geq \Delta$. One can easily show that the CB regions for the other even(odd) states are spread(squeezed) like that for $n = 0(1)$.

Adjacent to the CB region, we have a so-called "plateau region" where the tunneling current is dominated by the tunneling rate through one junction which behaves like a bottleneck of the tunneling current. In the plateau region the tunneling current is carried by the even and the odd states in the following manner. An electron with spin σ tunnels into the island from the left electrode. While staying in the island, the spin of the electron relaxes due to the spin orbit interaction, surface

scattering, and/or the hyperfine contact interaction [18]. After a certain time period determined by the tunneling rate $\Gamma^-_{2\sigma}(1)$, the electron tunnels out of the island. Therefore, no spin accumulation occurs even at the finite bias voltage. Outside the plateau region, continuous quasiparticle states contribute to the tunneling current and the spin accumulation can exist.

We consider the tunneling current and TMR in the plateau region, for example, indicated by shade in Fig. 1 (b), which is determined by $E_1^+(0) < -\Delta, E_1^+(1) > \Delta$, and $|E_2^-(1)| < \Delta$. In this region, the following three states are available: $|0\rangle$ three is no excess electron in the island, $|\uparrow\rangle$ there is one up-spin excess electron in the island, $|\downarrow\rangle$ there is one down-spin excess electron in the island. The transition rate from $|0\rangle$ to $|\sigma\rangle$ is given by

$$\Gamma^+_\sigma \equiv \Gamma^+_{1\sigma}(0) = \frac{1}{e^2 R_{1\sigma}} \sqrt{E_1^+(0)^2 - \Delta^2}, \tag{9}$$

and the transition rate from $|\sigma\rangle$ to $|0\rangle$ is

$$\Gamma^-_\sigma \equiv \Gamma^-_{2\sigma}(1) = \frac{d}{e^2 R_{2\sigma}}. \tag{10}$$

We also introduce the spin relaxation rate η which corresponds to the transition rate between $|\uparrow\rangle$ and $|\downarrow\rangle$.

In order to obtain the tunneling current, we construct the master equation for the probabilities of states p_0, $p_\uparrow$, and $p_\downarrow$, which is given by

$$\dot{p}_0 = \Gamma^-_\uparrow p_\uparrow + \Gamma^-_\downarrow p_\downarrow - \left(\Gamma^+_\uparrow + \Gamma^+_\downarrow\right) p_0 \tag{11}$$

$$\dot{p}_\sigma = \eta \left(p_{\bar{\sigma}} - p_\sigma\right) + \Gamma^+_\sigma p_0 - \Gamma^-_\sigma p_\sigma \tag{12}$$

with the normalization condition $p_0 + p_\uparrow + p_\downarrow = 1$. We calculate the stationary probabilities by requiring $\dot{p}_0 = \dot{p}_\uparrow = \dot{p}_\downarrow = 0$. The solutions are

$$p_0 = \frac{\eta(\Gamma^-_\uparrow + \Gamma^-_\downarrow) + \Gamma^+_\uparrow \Gamma^+_\downarrow}{\gamma} \tag{13}$$

$$p_\sigma = \frac{\eta\left(\Gamma^+_\sigma + \Gamma^+_{\bar{\sigma}}\right) + \Gamma^+_\sigma \Gamma^-_{\bar{\sigma}}}{\gamma}, \tag{14}$$

where $\gamma = \sum_{\sigma=\uparrow\downarrow} \eta\left(2\Gamma^+_\sigma + \Gamma^-_\sigma\right) + \Gamma^+_\sigma \Gamma^-_{\bar{\sigma}} + \frac{1}{2}\Gamma^-_\sigma \Gamma^-_{\bar{\sigma}}$ and the subscript $\bar{\sigma}$ represents the spin direction opposite to σ. The similar result has been obtained for the single discrete level system[19]. Knowing the stationary probabilities we can calculate the tunneling current through the left junction

$$I = -e p_0 \left(\Gamma^+_\uparrow + \Gamma^+_\downarrow\right). \tag{15}$$

The tunneling current of the F/S/F SET transistor given by Eq. (15) depends strongly on whether the magnetizations in ferromagnetic electrodes are parallel or antiparallel. For the ferromagnetic(F)-alignment, where the magnetizations are parallel, the junction resistances can be expressed as $R_{1\uparrow} = R_{2\uparrow} = R_M$ and $R_{1\downarrow} = R_{2\downarrow} = R_m$. Here R_M is the junction resistance for electrons in the majority spin band and $R_m = (1-P)/(1+P)R_M$ is for those in the minority spin band. We write the transition rates by using the subscripts M and m as $\Gamma^\pm_\uparrow = \Gamma^\pm_M$ and $\Gamma^\pm_\downarrow = \Gamma^\pm_m$. Introducing $\alpha \equiv R_m/R_M$, $\beta \equiv \Gamma^-_M/\Gamma^+_M, \zeta \equiv \eta/\Gamma^-_M$, the probability of the state $|0\rangle$ for the F-alignment is expressed by

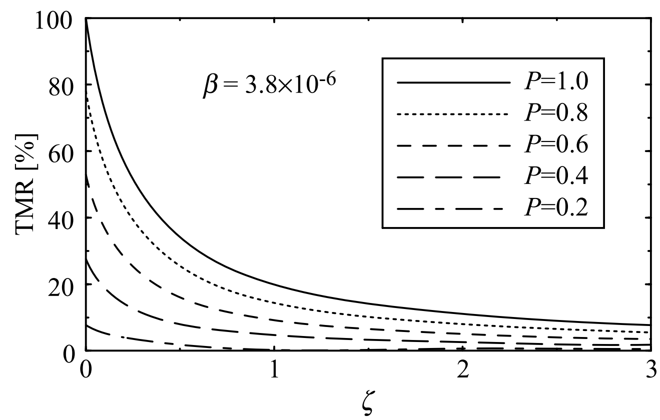

FIG. 2: TMR of the F/S/F SET with $\beta = 3.8 \times 10^{-6}$ is plotted against the normalized spin relaxation rate ζ. From top to bottom: $P = 1.0, 0.8, 0.6, 0.4$, and 0.2.

$$p_0^F = \frac{\zeta\beta(\alpha+1) + \alpha\beta}{\zeta(\alpha+1)(\beta+2) + \alpha(\beta+2)} = \frac{\beta}{\beta+2}. \tag{16}$$

The probability p_0^F is independent of the spin relaxation rate. For the antiferromagnetic(A)-alignment, where the magnetizations are antiparallel, the probability of the even state is

$$p_0^A = \frac{\zeta\beta(\alpha+1) + \alpha\beta}{\zeta(\alpha+1)(\beta+2) + \alpha^2 + \alpha\beta + 1}. \tag{17}$$

The TMR is calculated by the formula $TMR = 1 - I_A/I_F$. From Eqs. (15)- (17), we have

$$TMR = \frac{(\alpha-1)^2}{\zeta(\alpha+1)(\beta+2) + \alpha^2 + \alpha\beta + 1}. \tag{18}$$

The value β is much smaller than unity in almost all area of the plateau region since $d \ll E_C$. If we assume that Δ=0.18 meV (Aluminum), E_C=8.0meV and $d = 3.0 \times 10^{-5}$ meV [4], then we have $\beta = 3.8 \times 10^{-6}$ at the center of the plateau region. In Figure 2, the TMR for $\beta = 3.8 \times 10^{-6}$ is plotted against the normalized spin relaxation rate ζ. The TMR is monotonically decreasing function of ζ and the spin relaxation rate of an excess electron η can be estimated from the TMR by fitting the experimental data. If there is no spin relaxation process in the island, $\zeta = 0$, the TMR is approximately given by $TMR \simeq 2P^2/(1+P^2)$.

The fact that the TMR depends on the spin relaxation rate η via its normalized value ζ means that how much the spin information is transmitted is determined by the

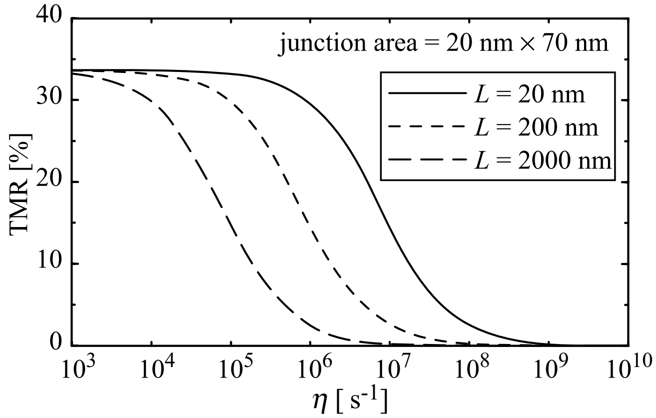

FIG. 3: TMR of the F/S/F SET with $P = 0.45$, which is the typical value for Co [20], is plotted against the spin relaxation rate η. From top to bottom: the length of the island is L= 20, 200, and 2000 nm. The area of the junction is fixed at 20nm × 70 nm[4] and the junction resistance $R_M = 1$ MΩ for L=2000 nm. We assume that Δ=0.18 meV (Aluminum), E_C=8.0meV and $d = 3.0 \times 10^{-5}$ meV and the working point is set to the center of the plateau region.

competition between the spin relaxation rate and the tunneling rate through the right junction. In the plateau region the inverse of the tunneling rate $\Gamma^-_{M(m)}$ describes how long the excess electron with majority(minority) spin stays in the island. Therefore, the normalized spin relaxation rate ζ represents the probability that the electron with the majority spin tunnels out of the island holding its spin direction and the TMR is a function of ζ.

In the WKB approximation, the value $|T|^2$ is inversely proportional to the length of the island L [21, 22]. If the junction parameters other than L are kept fixed, the density of states $N^I_\sigma(= 1/d)$ is proportional to L. In this situation, the tunneling rate through the left junction does not depend on L since the size dependences of $|T|^2$ and N^I_σ in R^1_σ cancel out. On the contrary, the tunneling rate through the right junction is inversely proportional to L. The normalized spin relaxation rate ζ decreases and therefore the TMR increases with decreasing L. In Fig. 3 the TMR of the F/S/F SET transistor with $P = 0.45$, which is the typical value for Co [20], is plotted against the spin relaxation rate η for various values of L. We assume that the area of the junction is fixed at 20 nm × 70 nm and L=2000 nm, 200, and 20 nm corresponding to the Aluminum island with the average level spacing $d = 3.0 \times 10^{-5}$, 3.0×10^{-4}, and 3.0×10^{-3} meV, respectively [4]. For the spin relaxation rate $\eta = 10^7$ s^{-1}, which is of the same order as that caused by the hyperfine contact interaction [18], the TMR is 0.26, 2.4, and 15 % for L = 2000, 200, and 20 nm, respectively.

Very recently, Chen *et al.* reported the experimental evidence for suppression of superconductivity by spin imbalance in Co/Al/Co SET transistors[23]. Although they observed the spin imbalance outside the plateau region, their experimental results show that the effect we proposed is relevant to current experiments.

In conclusion, we theoretically study the TMR of F/S/F SET transistors with $E_C > \Delta$. We show that in the plateau region, there is no spin accumulation even at the finite bias voltage. However, the information of the injected spin is carried by the excess electron and the TMR exists. The spin relaxation rate of the excess electron can be estimated from the TMR. We also found that the TMR increases with decreasing the size of the superconducting island.

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[1] H. Grabert and M. H. Devoret, eds., *Single Charge Tunneling*, vol. B 294 of *NATO ASI Series* (Plenum Press, New York, 1992).
[2] L. L. Sohn, L. P. Kouwenhoven, and G. Schön, eds., *Mesoscopic Electron Transport*, vol. E 345 of *NATO ASI Series* (Kluwer Academic Publishers, Netherlands, 1996).
[3] R. Fazio and G. Schön, p407 in Ref.[2].
[4] D. C. Ralph, C. T. Black, J. M. Hergenrother, J. G. Lu, and M. Tinkham, p447 in Ref.[2].
[5] D. V. Averin and Y. V. Nazarov, Phys. Rev. Lett. **69**, 1993 (1992).
[6] M. T. Tuominen, J. M. Hergenrother, T. S. Tighe, and M. Tinkham, Phys. Rev. Lett. **69**, 1997 (1992).
[7] T. M. Eiles, J. M. Martinis, and M. H. Devoret, Phys. Rev. Lett. **70**, 1862 (1993).
[8] Y. Utsumi, H. Imamura, M. Hayashi, and H. Ebisawa, Physica C **376**, 237 (2002).
[9] S. Maekawa and T. Shinjo, eds., *SPIN DEPENDENT TRANSPORT IN MAGNETIC NANOSTRACTURES*, vol. 3 of *Advances in Condensed Matter Science* (Taylor & Francis, London, 2002).
[10] M. Ziese and M. J. Thornton, eds., *Spin Electronics*, vol. LNP 569 of *Lecture Notes in Physics* (Springer-Verlag, Berlin, 2001).
[11] H. Imamura, S. Takahashi, and S. Maekawa, Phys. Rev. B **59**, 6017 (1999).
[12] A. Brataas, Y. V. Nazarov, J. Inoue, and G. E. W. Bauer, Phys. Rev. B **59**, 93 (1999).
[13] J. Barnaś and A. Fert, Europhys. Lett. **44**, 85 (1998).
[14] A. N. Korotkov and V. I. Safarov, Phys. Rev. B **59**, 89 (1999).
[15] S. Takahashi, H. Imamura, and S. Maekawa, Phys. Rev. Lett. **82**, 3911 (1999).
[16] Z. W. Dong, R. Ramesh, T. Venkatesan, M. Johnson, Z. Y. Chen, S. P. Pai, V. Talyansky, R. P. Sharma, R. Shreekala, C. J. Lobb, et al., Appl. Phys. Lett. **71**, 1718 (1997).

[17] F. W. J. Hekking, L. I. Glazman, K. A. Matveev, and R. I. Shekhter, Phys. Rev. Lett. **70**, 4138 (1993).
[18] Y. Yafet, Phys. Lett. **98A**, 287 (1983).
[19] W. Rudziński and J. Barnaś, Phys. Rev. B **64**, 085318 (2001).
[20] J. S. Moodera and G. Mathon, J. Magn. Magn. Mater **200**, 248 (1999).
[21] J. Bardeen, Phys. Rev. Lett. **6**, 57 (1961).
[22] W. A. Harrison, Phys. Rev. **123**, 85 (1961).
[23] C. D. Chen, W. Kuo, D. S. Chung, J. H. Shyu, and C. S. Wu, Phys. Rev. Lett. **88**, 047004 (2002).